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Exclusive decay of 1^{--} quarkonia and B_c meson into a lepton pair combined with two pions

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Abstract. We study the exclusive decay of J/Ψ, Υ and B_c into a lepton pair combined with two pions in two kinematic regions. One is specified by the two pions having large momenta, but a small invariant mass. The other is specified by the two pions having small momenta. In both cases we find that in the heavy quark limit the decay amplitude takes a factorized form, in which the non-perturbative effect related to the heavy meson is represented by a NRQCD matrix element. The non-perturbative effects related to the two pions are represented by some universal functions characterizing the conversion of gluons into pions. Using models for these universal functions and chiral perturbative theory we are able to obtain numerical predictions for the decay widths. Our numerical results show that the decay of J/ψ is of the order of 10^{-5} with reasonable cuts and can be observed at BES II and the proposed BES III and CLEO-C. For other decays the branching ratio may be too small to be measured.

1 Introduction

50×10^6 J/ψ events have been collected with the upgraded Bejing Spectrometer (BES II) at the Beijing Electron Positron Collider (BEPC), and several billions J/ψ events will be collected with the proposed BES III at BEPC II and CLEO-C at the modified Cornell Electron Storage Ring (CESR) [1,2]. Furthermore, about $4\,\mathrm{fb}^{-1}$ $b\bar{b}$ resonance data are planned to be taken at CLEO III in the year prior to conversion to low energy operation (CLEO-C) [2]. With these data samples various decay modes of J/ψ and $b\bar{b}$ resonances can be studied with high statistics. In this paper we propose to study the exclusive decay of 1^{--} quarkonia and B_c into a lepton pair and a pion pair. We consider two limited cases in the kinematic region. One is specified by the pion pair having a large total momentum and a small invariant mass. In this case, the pions are hard. The other is specified by the pion pair having a small momentum, i.e., the pions are soft. In these decays the two pion system is produced through conversion of gluons into the two pions. This is because isospin symmetry conversion of gluons into one pion is highly suppressed. In a two-pion system the two pions can be in an isospin singlet; then the conversion is allowed. Hence these decays will provide valuable information on how unobservable gluons, as dynamical degrees of freedom of QCD, are converted into observable hadrons.

In the case that the two-pion system has an invariant mass $m_{\pi\pi}$ which is much smaller than the mass of a heavy meson and has a large total momentum, the decay amplitude takes a factorized form in the heavy quark limit, in which the non-perturbative effect related to a heavy meson is represented by a non-relativistic QCD (NRQCD) matrix element [3], and that related to the two pions is represented by a distribution amplitude of two gluons in the isoscalar pion pair which is defined with twist-2 operators. The two gluons are hard in the kinematical region, their emission rate can be calculated with perturbative QCD. The same distribution amplitude also appears in the predictions for the production of two pions in the exclusive processes $\gamma+\gamma^* \to \pi+\pi$ [4–6], $\gamma^*+h \to h+\pi+\pi$ [7,8], and the radiative decay of 1^{--} heavy quarkonium [9], where the amplitudes can be factorized in a certain kinematic region. Besides these processes, the decays studied here will provide another way to study the non-perturbative mechanism of how gluons, which are fundamental dynamical freedoms of QCD, are transmitted into two pions. Furthermore, for $\gamma + \gamma^* \to \pi + \pi$, at the tree level, only the distribution amplitude of the quark appears in the scattering amplitude, the distribution amplitude of gluon appears at loop levels or through the evolution of the distribution amplitudes [4–6], while for $\gamma^* + h \to h + \pi + \pi$, at the tree level, the distribution amplitude of the quark as well as that of the gluon contribute to the scattering amplitude, but the produced charged pion pair is dominantly in an isospin $I = 1$ state [7,8]. This may make it difficult to extract the distribution amplitude of the gluon from corresponding experimental data, because the two pions produced through gluon conversion are in a $I = 0$ state. For the decays studied in this paper and the radiative decay of 1^{--} heavy quarknonium to two pions [9], only the distribution amplitude of the gluon appears at the tree level and the produced two pions are dominantly in a $I = 0$ state. This makes the extraction of the gluon content of a two-pion system relatively easier in experiment. Of course, comparing with the radiative decay of 1^{--} heavy quarkonium into two pions in the same kinematic region, the leptonic decay of 1^{--} heavy quarkonium to two pions is suppressed by the fine structure constant α, but the final state in the latter case is clearer and can be detected with a higher efficiency. With the model for the distribution amplitude of gluon developed in [6,8], we obtain numerical predictions for the branch ratio of the decay in the considered kinematic region. Our results show that the decay mode of J/ψ in the considered kinematic region is certainly observable at BES II and the proposed BES III and CLEO-C. For other decays the branching ratios may be too small to be measured.

In the case with two soft pions, it has been shown that the decay amplitude of J/Ψ and of Υ also takes a factorized form in the heavy quark limit [10]. In the decay amplitude, the non-perturbative effect related to heavy quarkonium and that related to a pion pair can be separated; the former is still represented by a NRQCD matrix element, while the latter is represented by the matrix element of a correlator of electric chromofields which characterizes the soft gluon transition into the pion pair. This result is non-perturbative. For B_c decay one can generalize the approach and obtain a factorized form for the decay amplitude, where the same correlator appears. Since the matrix element of the correlator of electric chromofields between the vacuum state and the two-pion state is unknown, no numerical prediction of the decay is given in [10]. In this paper, we develop a model for the matrix element of the correlator of electric chromofields and give numerical predictions for the leptonic decays $J/\psi, \Upsilon(1S)$ and B_c into a soft pion pair. Numerical results are obtained in the considered region of kinematics and show that the decay mode of J/ψ is observable at BES II and at the proposed BES III and CLEO-C.

This paper is organized as follows. In Sect. 2 we study the decays of J/ψ and $\Upsilon(1S)$ into two hard pions combined with a lepton pair, where we give a detailed derivation of the factorized amplitude of the decay. Numerical results for the decays are presented. In Sect. 3 the decays of J/ψ and $\Upsilon(1S)$ into two soft pions combined with a lepton pair are studied, a model for the matrix element of the correlator of electric chromofields is developed, and numerical results for the decay are given. Section 4 is devoted to the study of the decays of B_c. We summarize our work in Sect. 5.

In this paper, we take a non-relativistic normalization for the heavy meson states and for heavy quarks, and we take the pion pair to be a π^+ and a π^-. Using isospin symmetry one can easily obtain results for a pair of π^0's.

2 Leptonic decays of J/ψ and $\Upsilon(1S)$ to two hard pions

We study the exclusive decay in the rest frame of J/ψ:

$$J/\psi(P) \to l^+(p_1) + l^-(p_2) + \pi^+(k_{\pi^+}) + \pi^-(k_{\pi^-}), \quad (1)$$

where $l = e, \mu$, and the momenta are indicated in the brackets. We denote $k = k_{\pi^+} + k_{\pi^-}, q = p_1 + p_2$ and $m_{\pi\pi}^2 = k^2$. We consider the kinematic region where $|\boldsymbol{k}| \gg m_{\pi\pi}$ and $m_{\pi\pi} \ll M_\psi$. At leading order of QED, the S-matrix element for the decay is

$$\langle f|S|i\rangle = -\mathrm{i}Q_c e^2 L_\mu \cdot \frac{1}{q^2}\int \mathrm{d}^4 z \mathrm{e}^{\mathrm{i}q\cdot z}\langle \pi^+\pi^-|\bar{c}(z)\gamma^\mu c(z)|J/\psi\rangle, \quad (2)$$

where Q_c is the electric charge of the c-quark in units of e, $c(z)$ is the Dirac field for the c-quark, and

$$L_\mu = \bar{u}(p_2)\gamma_\mu v(p_1). \quad (3)$$

At leading order of QCD, two gluons are emitted by the c- or $\bar{c}$-quark, and these two gluons will be transmitted into the two pions. Using the Wick theorem we obtain

$$\begin{aligned}\langle f|S|i\rangle &= \frac{\mathrm{i}}{2}\frac{1}{2}\delta^{ab}Q_c e^2 g_s^2 L_\mu \cdot \frac{1}{q^2} \\ &\times \int \mathrm{d}^4 z \mathrm{d}^4 y \mathrm{d}^4 x \mathrm{d}^4 x_1 \mathrm{d}^4 y_1 \mathrm{e}^{\mathrm{i}(q\cdot z + k_2\cdot y)} \\ &\times \langle 0|\bar{c}_j(x_1)c_i(y_1)|J/\psi\rangle\langle\pi^+\pi^-|G^a_{\mu_1}(x)G^b_{\nu_1}(0)|0\rangle \\ &\times \Big[\delta^4(x-x_1)\delta^4(z-y_1)\gamma^{\mu_1} \\ &\times S_\mathrm{F}(x-y)\gamma^{\nu_1}S_\mathrm{F}(y-z)\gamma^\mu + \cdots\Big]_{ji}, \end{aligned} \quad (4)$$

where k_2 is the momentum of one of the emitted gluons, $S_\mathrm{F}(x-y)$ is the Feynman propagator of the c-quark, and the dots in the square brackets denote the other five terms. In the limit of $m_c \to \infty$, a c- or $\bar{c}$-quark moves with a small velocity v; this fact enables us to describe the non-perturbative effect related to J/ψ by NRQCD [3]. For the matrix element $\langle 0|\bar{c}_j(x_1)c_i(y_1)|J/\psi\rangle$ the expansion in v can be performed with the result

$$\begin{aligned}&\langle 0|\bar{c}_j(x_1)c_i(y_1)|J/\psi\rangle \\ &= -\frac{1}{6}(P_+\gamma^\ell P_-)_{ij}\langle 0|\chi^\dagger\sigma^\ell\psi|J/\psi\rangle \mathrm{e}^{-\mathrm{i}p\cdot(x_1+y_1)} + \mathrm{O}(v^2),\end{aligned} \quad (5)$$

where $\chi^\dagger(\psi)$ is the NRQCD field for the $\bar{c}(c)$-quark, $\sigma^\ell(\ell = 1,2,3)$ is the Pauli matrix, and

$$P_\pm = \frac{1}{2}(1\pm\gamma^0), \qquad p = (m_c, 0, 0, 0). \quad (6)$$

The matrix $\langle 0|\chi^\dagger\sigma^\ell\psi|J/\psi\rangle$ is proportional to the polarization vector $\varepsilon^\ell(J/\psi)$ at the considered order. In this paper, we neglect the contribution from higher orders in v; the momentum of J/ψ is then approximated by $2p$. It should be noted that effects at higher order of v can be added with the expansion in (5).

Using (5) we can write the S-matrix element as

$$\begin{aligned}\langle f|S|i\rangle &= \frac{-\mathrm{i}}{24}Q_c e^2 g_s^2 (2\pi)^4\delta^4(2p-k-q)L_\mu \\ &\times \frac{1}{q^2}\langle 0|\chi^\dagger\sigma^\ell\psi|J/\psi\rangle \\ &\times \int\frac{\mathrm{d}^4 k_1}{(2\pi)^4}H^{\ell\mu\mu_1\nu_1}(p,k,k_1)\Gamma_{\mu_1\nu_1}(k,k_1),\end{aligned} \quad (7)$$

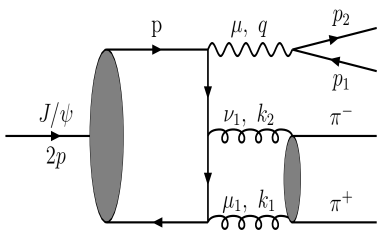

Fig. 1. One of the Feynman diagrams for the exclusive decay of J/ψ into lepton pair and two pions

and

$$\Gamma^{\mu\nu}(k,k_1) = \int \mathrm{d}^4 x \mathrm{e}^{-\mathrm{i}k_1 x} \langle \pi^+\pi^- | G^{a,\mu}(x) G^{a,\nu}(0)|0\rangle, \quad (8)$$

where $H^{\ell\mu\mu_1\nu_1}(p,k,k_1)$ is the amplitude for a $c\bar{c}$ pair emitting a virtual photon and two gluons, and this can be calculated with perturbative QCD. The contributions in (7) can be represented by Feynman diagrams. One of them is given in Fig. 1, where the kinematic variables are also indicated. The non-perturbative object $\Gamma^{\mu\nu}(k,k_1)$ describes how two gluons are converted into two pions.

If the two-pion system has a large momentum and a small invariant mass, a twist expansion for the non-perturbative object $\Gamma^{\mu\nu}(k,k_1)$ can be performed. For convenience we will work in the light-cone coordinate system, in which the components of k are given by

$$k^\mu = (k^+, k^-, \mathbf{0}), \quad k^+ = (k^0 + k^3)/\sqrt{2},$$
$$k^- = (k^0 - k^3)/\sqrt{2}. \quad (9)$$

In the light-cone coordinate system we introduce two light-cone vectors and a tensor:

$$n^\mu = (0,1,0,0), \quad \tilde{n}^\mu = (1,0,0,0),$$
$$d_{\mathrm{T}}^{\mu\nu} = g^{\mu\nu} - n^\mu \tilde{n}^\nu - n^\nu \tilde{n}^\mu, \quad (10)$$

and we take the gauge

$$n \cdot G(x) = 0. \quad (11)$$

The x-dependence of the matrix element in (8) is controlled by different scales. The x^--dependence is controlled by k^+, which is large in the kinematic region we considered, while the x^+- and $\boldsymbol{x}_{\mathrm{T}}$-dependence are controlled by the scale k^- and Λ_{QCD}, which are small in comparison with k^+. With this observation we can expand the matrix element in x^+ and in $\boldsymbol{x}_{\mathrm{T}}$. The resulting twist expansion of the Fourier transformed matrix element $\Gamma^{\mu,\nu}(k,k_1)$ is a collinear expansion in k^- and $k_{\mathrm{T}} \sim \Lambda_{\mathrm{QCD}}$. Hence the expansion parameters of $\Gamma^{\mu,\nu}(k,k_1)$ are k^-/k^+ and $\Lambda_{\mathrm{QCD}}/k^+$, with $k^-/k^+ \leq 0.10$ and $\Lambda_{\mathrm{QCD}}/k^+ \approx 0.20$ for J/ψ in the kinematic region considered. At the leading order only twist-2 operators contribute to the matrix element. We will neglect higher orders in the expansion, i.e., we only keep contributions of twist-2 operators. Then we obtain

$$\Gamma^{\mu\nu}(k,k_1) = (2\pi)^4 \delta(k_1^-) \delta^2(k_{1T}) \frac{1}{k^+} \frac{1}{x_1(1-x_1)}$$
$$\times \left[\frac{1}{2} d_{\mathrm{T}}^{\mu\nu} \Phi^G(x_1, \zeta, m_{\pi\pi})\right], \quad (12)$$

with

$$\Phi^G(x_1,\zeta,m_{\pi\pi})$$
$$= \frac{1}{k^+} \int \frac{\mathrm{d}x^-}{2\pi} \mathrm{e}^{-\mathrm{i}k_1^+ x^-} \langle \pi^+\pi^- | G^{a,+\mu}(x^- n) G_\mu^{a,+}(0)|0\rangle,$$
$$x_1 = \frac{k_1 \cdot n}{k \cdot n}, \quad \zeta = \frac{k_+ \cdot n}{k \cdot n}. \quad (13)$$

$\Phi^G(x_1,\zeta,m_{\pi\pi})$ is the gluonic distribution amplitude which describes how a pion pair with helicity $\lambda = 0$ is produced by two collinear gluons; this represents a non-perturbative effect and can only be calculated with non-perturbative methods or extracted from experiment. As it stands, it is gauge invariant in the gauge $n \cdot G(x) = 0$. In other gauges we need to supply a Wilson line operator to make it gauge invariant. With (12) it is straightforward to obtain the S-matrix element at the tree level in our approximation:

$$\langle f|S|i\rangle = \frac{-\mathrm{i}}{24} Q_c e^2 g_s^2 (2\pi)^4 \delta^4(2p-k-q) L^\mu$$
$$\times \frac{1}{q^2} \langle 0|\chi^\dagger \sigma^\ell \psi | J/\psi\rangle$$
$$\times \int_0^1 \mathrm{d}x_1 \frac{\Phi^G(x_1,\zeta,m_{\pi\pi})}{x_1(1-x_1)}$$
$$\cdot \left[\frac{1}{2} d_{\mathrm{T}}^{\mu_1\nu_1} \cdot H_{\ell\mu\mu_1\nu_1}(p,k,k_1)\right], \quad (14)$$

with

$$\frac{1}{2} d_{\mathrm{T}}^{\mu_1\nu_1} \cdot H_{\ell\mu\mu_1\nu_1}(p,k,k_1) = \frac{16}{M_\psi^2} \tilde{n}_\mu n_\ell - \frac{16}{M_\psi^2 - q^2} g_{\mu\ell}, \quad (15)$$

where M_ψ is the mass of J/ψ. In (15) we have neglected the mass $m_{\pi\pi}$, since the effect of $m_{\pi\pi}$ should be combined with the effects of twist-4 operators as a correction to the above result. In (14) the non-perturbative effect related to J/ψ and that to the two-pion system are separated; the former is represented by a NRQCD matrix element, while the latter is represented by the distribution amplitude of two gluons in the isoscalar pion pair, which is defined in (13) in the gauge $n \cdot G(x) = 0$.

The kinematics of the decay can be fully described by five variables as in K_{e4} decay [11]:

(1) $m_{\pi\pi}^2$, the invariant mass squared of the pion pair;
(2) $q^2 = (p_1 + p_2)^2$, the invariant mass squared of the lepton pair;
(3) θ_π, the polar angle of the π^+ in the rest frame of the pion pair with respect to the moving direction of the pion pair in the J/ψ rest frame;
(4) θ_l, the polar angle of the l^+ in the rest frame of lepton pair with respect of the moving direction of the lepton pair in the J/ψ rest frame;
(5) ϕ, the azimuthal angle between the two planes in which the pion pair and the lepton pair lies respectively.

In terms of these variables, the differential decay width can be written as

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^8} \cdot \frac{\pi^2}{32} \cdot \frac{|\boldsymbol{k}|}{M_\psi}$$

$$\cdot \beta\beta_l \overline{\sum}|M|^2 \mathrm{d}q^2 \mathrm{d}m^2_{\pi\pi} \mathrm{d}\cos\theta_\pi \mathrm{d}\cos\theta_l \mathrm{d}\phi, \quad (16)$$

where β and β_l are defined by

$$\beta = \sqrt{1 - \frac{4m_\pi^2}{m_{\pi\pi}^2}}, \quad \beta_l = \sqrt{1 - \frac{4m_l^2}{q^2}}, \quad (17)$$

$\overline{\sum}|M|^2$ is the absolute squared matrix element of the decay, summed over final state spins and averaged over the initial state spin. From (14) and (15), we have

$$\overline{\sum}|M|^2 = \frac{1}{24^2} Q_c^2 e^4 g_s^4 \cdot \frac{1}{q^4} \cdot |\langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|J/\psi\rangle|^2 \times \frac{512 q^2[(M_\psi^2 + q^2) + (M_\psi^2 - q^2)\cos^2\theta_l]}{3M_\psi^2 (M_\psi^2 - q^2)^2} \times \left| \int_0^1 \mathrm{d}x_1 \frac{\Phi^G(x_1, \zeta, m_{\pi\pi})}{x_1(1-x_1)} \right|^2, \quad (18)$$

which is independent of the azimuthal angle ϕ; the spin average for J/ψ is implied in the squared matrix element.

To give numerical predictions, the non-perturbative inputs, the NRQCD matrix element and the distribution amplitude of two gluons in the isoscalar pion pair, are needed. The NRQCD matrix element is related to the wave-function of J/ψ in potential models and can be estimated with these models. It can also be calculated with lattice QCD or extracted from experiment. In this paper, we use leptonic decay of J/ψ to determine the NRQCD matrix element, i.e.,

$$\Gamma(J/\psi \to e^+e^-) = \frac{2\pi Q_c^2 \alpha^2}{3m_c^2} \cdot |\langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|J/\psi\rangle|^2. \quad (19)$$

The distribution amplitude $\Phi^G(x_1, \zeta, m_{\pi\pi})$ is not determined by experiment; a detailed study of $\Phi^G(x_1, \zeta, m_{\pi\pi})$ can be found in [5,6,8]. For our numerical prediction we will use these results for $\Phi^G(x_1, \zeta, m_{\pi\pi})$, in which the asymptotic form of $\Phi^G(x_1, \zeta, m_{\pi\pi})$ is taken as an Ansatz for $\Phi^G(x_1, \zeta, m_{\pi\pi})$. It should be noted that the renormalization scale μ should be taken as M_ψ in our case. Because it is not very large, the actual shape of $\Phi^G(x_1, \zeta, m_{\pi\pi})$ may look dramatically different from that of the asymptotic form. Keeping this in mind we take the form $\Phi^G(x_1, \zeta, m_{\pi\pi})$ as that given in [8]:

$$\Phi^G(x_1, \zeta, m_{\pi\pi}) = -60 M_2^G x_1^2 (1-x_1)^2 \times \left[\frac{3C - \beta^2}{12} f_0(m_{\pi\pi}) P_0(\cos\theta_\pi) - \frac{\beta^2}{6} f_2(m_{\pi\pi}) P_2(\cos\theta_\pi) \right], \quad (20)$$

where ζ is related to θ_π and $m_{\pi\pi}$ by

$$\beta\cos\theta_\pi = 2\zeta - 1. \quad (21)$$

C is a constant and takes values $C = 1 + bm_\pi^2 + O(m_\pi^4)$ with $b \simeq -1.7\,\mathrm{GeV}^{-2}$ [6,8], M_2^G is determined by gluon fragmentation into a single pion; its asymptotic value is

$$M_2^G = \frac{4C_\mathrm{F}}{N_f + 4C_\mathrm{F}}. \quad (22)$$

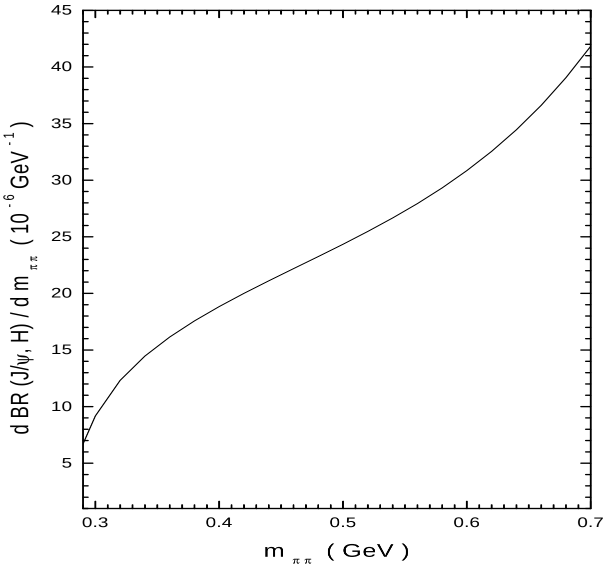

Fig. 2. The differential decay branching ratio of J/ψ, dBR $(J/\psi, H)/\mathrm{d}m_{\pi\pi}$, as a function of $m_{\pi\pi}$ in units of $10^{-6}\,\mathrm{GeV}^{-1}$ with the cuts given in the text

$f_0(m_{\pi\pi})$ and $f_2(m_{\pi\pi})$ are the Omnès functions for $I=0$ s- and d-wave $\pi\pi$ scattering, respectively. The Omnès function $f_2(m_{\pi\pi})$ is dominated by the $f_2(1270)$ resonance resulting in a peak at $m_{\pi\pi} = 1.275\,\mathrm{GeV}$, while the Omnès function $f_0(m_{\pi\pi})$ in the relevant $m_{\pi\pi}$ region we studied, $(m_{\pi\pi} \le 0.70\,\mathrm{GeV})$, can be calculated by the chiral perturbative theory. The result is [12]

$$f_0(m_{\pi\pi}) = 1 + \frac{m_{\pi\pi}^2}{192\pi^2 f_\pi^2} + \frac{2m_{\pi\pi}^2 - m_\pi^2}{32\pi^2 f_\pi^2} \times \left[\beta \ln\left(\frac{1-\beta}{1+\beta}\right) + 2 + \mathrm{i}\pi\beta \right], \quad (23)$$

where $f_\pi = 93\,\mathrm{MeV}$ is the pion decay constant.

Cuts must be used to select the kinematic region where the two-pion system has a large momentum and a small invariant mass. We use the cuts $k^+ \ge 10k^-$, $k^0 + |\boldsymbol{k}| \ge 2.0\,\mathrm{GeV}$ and $2m_\pi \le m_{\pi\pi} \le 0.70\,\mathrm{GeV}$; this corresponds to $q^2 \le 2.5\,\mathrm{GeV}^2$ for J/ψ and $q^2 \le 67\,GeV^2$ for $\Upsilon(1S)$.

With these results we are able to predict the decay branching ratio in the region considered. The quark masses are take as $m_c = (1/2)M_\psi$ and $m_b = (1/2)M_\Upsilon$. $\alpha_\mathrm{s}(2m_c) = 0.31$ for J/ψ, $\alpha_\mathrm{s}(2m_b) = 0.21$ for $\Upsilon(1S)$. All other parameters needed are taken from [14]. The $m_{\pi\pi}$ distribution of J/ψ decay, integrated over $|\cos\theta_\pi| \le 1.0$, $|\cos\theta_l| \le 1.0, 0 \le \phi \le 2\pi$, and $4m_l^2 \le q^2 \le 2.5\,\mathrm{GeV}^2$, denoted by $\mathrm{dBR}(J/\psi, H)/\mathrm{d}m_{\pi\pi}$, is shown in Fig. 2. The q^2 distribution of J/ψ decay, integrated over $|\cos\theta_\pi| \le 1.0, |\cos\theta_l| \le 1.0, 0 \le \phi \le 2\pi$, and $2m_\pi \le m_{\pi\pi} \le 0.70$ GeV, denoted by $\mathrm{dBR}(J/\psi, H)/\mathrm{d}q^2$, is shown in Fig. 3. The q^2 distribution decreases rapidly as q^2 increases; this is mainly due to the q^{-2} factor of the photon propagator. This behavior is shown in another way in Fig. 4, where the decay branching ratio as a function of the cut M^2 with $q^2 < M^2$ is shown. We see from this figure, for $M^2 = 10^{-2}, 10^{-1}, 10^0\,\mathrm{GeV}^2$, that the corresponding con-

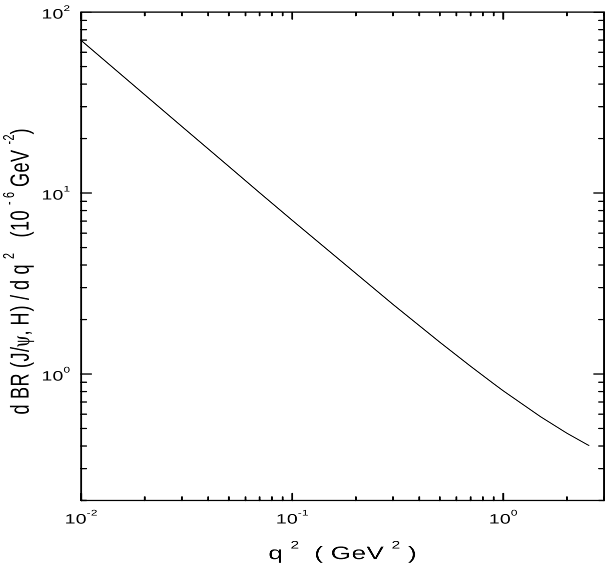

Fig. 3. The differential decay branching ratio of J/ψ, dBR $(J/\psi, H)/\mathrm{d}q^2$ as a function of q^2 in unit of $10^{-6}\,\mathrm{GeV}^{-2}$ with the cuts given in text

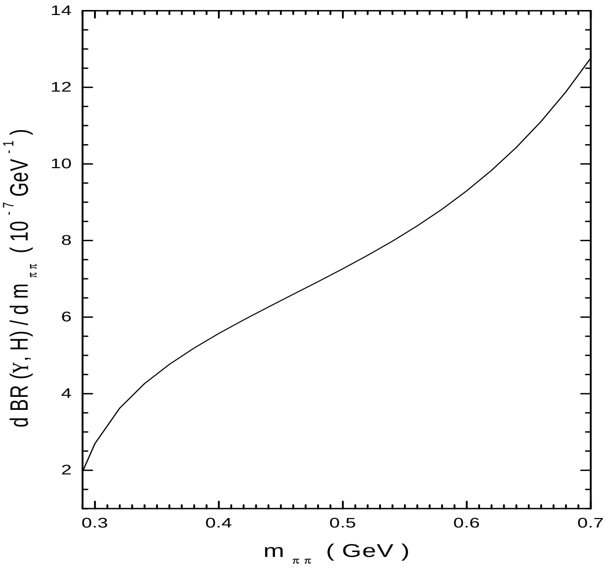

Fig. 5. The differential decay branching ratio of $\Upsilon(1S)$, dBR $(\Upsilon, H)/\mathrm{d}m_{\pi\pi}$, as a function of $m_{\pi\pi}$ in units of $10^{-7}\,\mathrm{GeV}^{-1}$ with the cuts given in the text

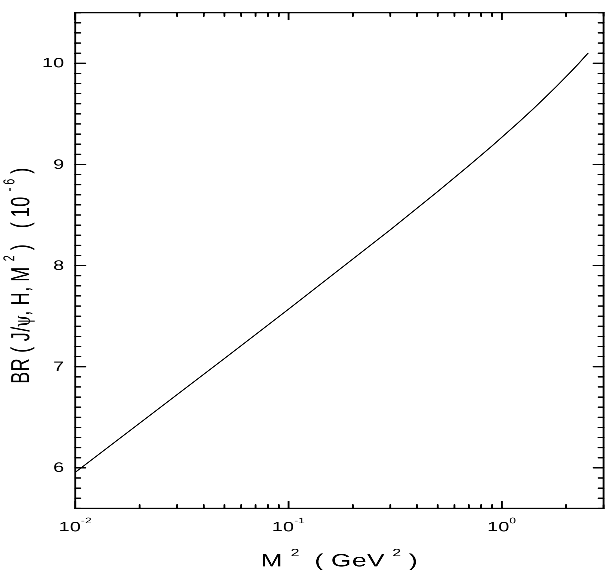

Fig. 4. The decay branching ratio of J/ψ, BR$(J/\psi, H, M^2)$, as a function of M^2 with regel580 $M^2 > q^2$ in units of 10^{-6}. The other cuts are the same

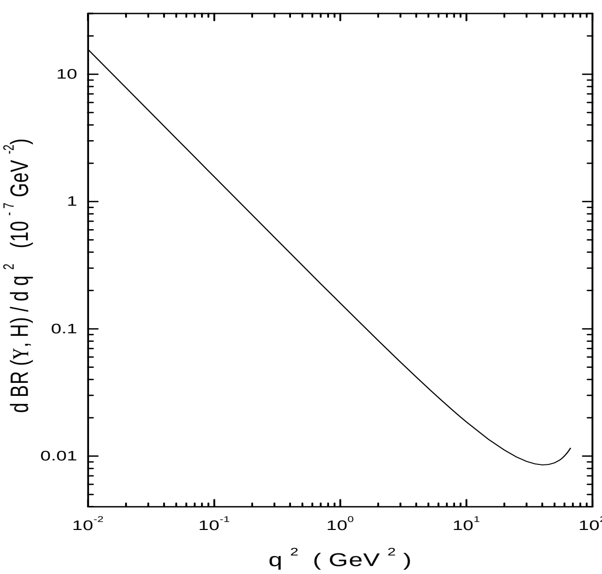

Fig. 6. The differential decay branching ratio of $\Upsilon(1S)$, dBR $(\Upsilon, H)/\mathrm{d}q^2$ as a function of q^2 in units of $10^{-7}\,\mathrm{GeV}^{-2}$ with the cuts given in the text

tributions are $59\%, 75\%, 92\%$ to the branching ratio in the whole region considered, respectively. Integrating over the kinematic region we considered, the decay branching ratio is 1.0×10^{-5} for J/ψ, among which the s-wave and d-wave contributions are 9.2×10^{-6} and 8.6×10^{-7} respectively, i.e., the d-wave contribution is suppressed in the kinematic region here. The results indicate that this decay mode as well as the distributions of $m_{\pi\pi}$ and q^2 can be observed at BES II, and at the proposed BES III and CLEO-C. In the above numerical calculations, the renormalization scale of the effective QCD coupling is taken to be J/ψ mass with $\Lambda^{(4)}_{\mathrm{QCD}} = 280\,\mathrm{MeV}$ [14], i.e., $\alpha_{\mathrm{s}}(2m_c) = 0.31$. If this scale is taken to be m_c, the corresponding decay branching ratio of J/ψ in the considered kinematic region increases by a factor of 2.1 by using (18). Hence, the decay branching ratio is conservatively predicted.

The corresponding differential decay branching ratios for $\Upsilon(1S)$ decay are shown in Figs. 5–7. The kinematic region we studied for $\Upsilon(1S)$ decay is $|\cos\theta_\pi| \le 1.0, |\cos\theta_l| \le 1.0$, $0 \le \phi \le 2\pi$, $2m_\pi \le m_{\pi\pi} \le 0.70\,\mathrm{GeV}$, and $4m_l^2 \le q^2 \le 67\,\mathrm{GeV}^2$. It is interesting to observe that in Fig. 6 there is a turn-over near $q^2 = 40\,\mathrm{GeV}^2$. Similar to the J/ψ case, the dominant contribution to the decay of $\Upsilon(1S)$ comes also from the small q^2 region. For $M^2 = 10^{-2}, 10^{-1}$,

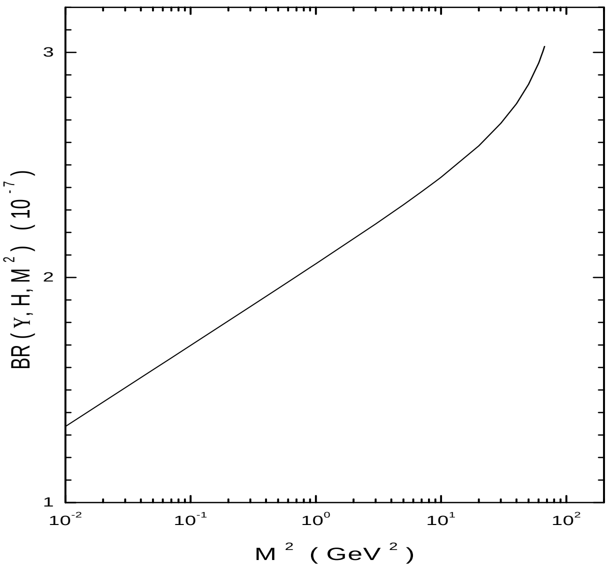

Fig. 7. The decay branching ratio of $\Upsilon(1S)$, $\mathrm{BR}(\Upsilon, H, M^2)$, as a function of M^2 with $M^2 > q^2$ in units of 10^{-7}. The other cuts are the same

10^0, and $10^1\,\mathrm{GeV}^2$, the corresponding contributions are $44\%, 56\%, 68\%$, and 81% to the branching ratio in the whole region considered, respectively. Integrating over the kinematic region we considered, the decay branching ratio is 3.0×10^{-7}, among which the s-wave and d-wave contributions are 2.8×10^{-7} and 2.6×10^{-8} respectively, i.e., the d-wave contribution is also suppressed in the kinematic region here. For the decay of Υ one may allow $m_{\pi\pi}$ to be larger than that in the decay of J/ψ, i.e., $0.7\,\mathrm{GeV}$, because the phase space is large. With a large upper cut for $m_{\pi\pi}$, the branching ratio can become large. But $f_0(m_{\pi\pi})$ determined with chiral perturbation theory may become unreliable for large $m_{\pi\pi}$.

It should be noted that the two-pion state is produced with the helicity $\lambda = 0$ at the level of leading twist. It can be a mixture of states with different angular momenta L. Because of parity conservation, isospin symmetry and Bose–Einstein statistics, L can only be even. Our numerical results show that the state is mainly in a s-wave state. At levels of higher twist it is possible that the two-pion state is produced with $\lambda \neq 0$. Following the analysis for the radiative decay of Υ into $f_2(1270)$ [13], one can show that the two-pion state can be produced with $|\lambda| = 1$ and 2 at the order of twist 3 and of twist 4, respectively.

3 Leptonic decays of J/ψ and $\Upsilon(1S)$ combined with two soft pions

In this section, we study the leptonic decays of J/ψ and $\Upsilon(1S)$ combined with two soft pions. In contrast to the decays studied in the last section the gluons, which are emitted by the heavy quarks in the quarkonium and are converted into the pions, are soft. The emission of soft gluons can be studied by employing an expansion in the inverse of the heavy quark mass m_Q. It is shown in [10] that at leading order of the expansion the decay amplitude in this kinematic region can be factorized into three parts: the first part is a NRQCD matrix element representing the bound-state effect of heavy quarkonium, the second part is a matrix element of a correlator of electric chromofields, which indicates the non-perturbative effect of the soft gluons converted into the soft pion pair, and the third part consists of some coefficients. It should be emphasized that the results can be derived without using perturbative QCD. In this section we present a model for the matrix element of the correlator of electric chromofields, and give numerical predictions. The S-matrix for the J/ψ decay is [10]

$$\begin{aligned}\langle f|S|i\rangle &= \mathrm{i}\frac{2}{3}Q_c e^2 (2\pi)^4 \delta^4(2p-k-q) L_\mu \\ &\times \frac{g^{\mu\ell}}{q^2}\langle 0|\chi^\dagger \sigma^\ell \psi|J/\psi\rangle \\ &\times \frac{1}{m_c}\cdot\frac{1}{(k^0)^2}\cdot T_{\pi\pi}(k) + \mathcal{O}\left(\frac{1}{m_c^2}\right) + \mathcal{O}(v^2), \\ T_{\pi\pi}(k) &= \int \mathrm{d}\tau \frac{1}{1+\tau-\mathrm{i}0^+}\cdot\frac{1}{1-\tau-\mathrm{i}0^+} h(\tau,k), \quad (24)\end{aligned}$$

where the momenta are denoted in the same way as in the last section. For soft pions we have $|\boldsymbol{k}| \ll m_Q$ and $m_{\pi\pi} \ll m_Q$. $h(\tau, k)$ is the distribution amplitude for the soft gluons converted into two soft pions. It is defined by

$$\begin{aligned}h(\tau,k) &= \frac{g_s^2}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}t \mathrm{e}^{\mathrm{i}\tau k^0 t}\langle \pi^+\pi^-|\boldsymbol{E}^a(t,\boldsymbol{0}) \quad (25)\\ &\times \left[P\exp\left\{-\mathrm{i}g_s\int_{-t}^{t}\mathrm{d}x^0 G^{0,c}(x^0,\boldsymbol{0})\tau^c\right\}\right]_{ab}\boldsymbol{E}^b(-t,\boldsymbol{0})|0\rangle,\end{aligned}$$

where P denotes path-ordering and τ^c is the generator of SU(3) in the adjoint representation, $(\tau^c)_{ab} = -\mathrm{i}f_{abc}$. Because of energy conservation $h(\tau, k) = 0$ if $|\tau| > 1$. The term with $g^{\mu,\ell}$ in (24) is expected in the heavy quark limit. In this limit the emitted gluons will not change the spin of the heavy quarks; hence the spin of J/ψ is transferred to the virtual photon. Therefor the helicity of the two-pion state is zero.

The function $h(\tau, k)$ is unknown. We make an Ansatz for the τ-dependence in the function, and this Ansatz is motivated by the results used in the last section. We assume

$$h(\tau,k) = a(k)(1-\tau)^2(1+\tau)^2; \quad (26)$$

the function $a(k)$ can be obtained by integrating $h(\tau, k)$ over τ, and we obtain

$$a(k) = \frac{15\pi}{4k^0}\langle\pi^+\pi^-|\alpha_\mathrm{s}\boldsymbol{E}^a(0)\cdot\boldsymbol{E}^a(0)|0\rangle, \quad (27)$$

hence for $|\tau| \leq 1$,

$$h(\tau,k) = \frac{15\pi}{4k^0}(1-\tau)^2(1+\tau)^2\cdot\langle\pi^+\pi^-|\alpha_\mathrm{s}\boldsymbol{E}^a(0)\cdot\boldsymbol{E}^a(0)|0\rangle. \quad (28)$$

The matrix element of local chromoelectric fields $\langle\pi^+\pi^-|\alpha_s \boldsymbol{E}^a(0)\cdot\boldsymbol{E}^a(0)|0\rangle$, which appears in the decay amplitude of $\Psi' \to J/\psi\pi^+\pi^-$ in the QCD multipole expansion method [15–18], can be written in our notation [17]

$$\begin{aligned}&\langle\pi^+\pi^-|\alpha_s \boldsymbol{E}^a(0)\cdot\boldsymbol{E}^a(0)|0\rangle\\&=\frac{2\pi}{9}\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle+\langle\pi^+\pi^-|\alpha_s(\mu)\theta^G_{00}(\mu)|0\rangle\\&=\frac{2\pi}{9}\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle-\frac{1}{3}\alpha_s(\mu)M_2^G(\mu)(k^0)^2\left(1+\frac{2m_\pi^2}{m_{\pi\pi}^2}\right)\\&\times P_0(\cos\theta_\pi)+\frac{1}{3}\alpha_s(\mu)M_2^G(\mu)|\boldsymbol{k}|^2\beta^2P_2(\cos\theta_\pi),\qquad(29)\end{aligned}$$

where $\theta_{\mu\nu}$ is the total energy-momentum tensor of QCD, $\theta^G_{\mu\nu}$ is the gluonic component, $M_2^G(\mu)$ is determined by the gluon fragmentation into one pion as before. In [17], including $\mathcal{O}(m_\pi^2)$ corrections, $\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle = q^2 + 2m_\pi^2$ is obtained from some general considerations. This coincides with the result of chiral perturbation theory at leading order of the chiral expansion. Since the kinematic region we considered is only part of the whole phase space and $m_{\pi\pi}$ is not very near the $\pi^+\pi^-$ threshold, we expect that the correction from next-to-leading order of chiral perturbation theory is important, so we use the expression derived from chiral perturbation theory at next-to-leading order for $\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle$, i.e. [12]

$$\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle=(m_{\pi\pi}^2+2m_\pi^2)f_0(m_{\pi\pi})+b_\theta m_{\pi\pi}^4,\qquad(30)$$

with $b_\theta = 2.7\,\mathrm{GeV}^{-2}$.

With these results, we are able to predict the shape of the differential decay branching ratio numerically. We use the cuts $0 \le |\boldsymbol{k}| \le (1/10)M_\psi$ and $2m_\pi < m_{\pi\pi} < 0.7\,\mathrm{GeV}$ to make the pions soft. In Fig. 8 the differential decay branching ratio of J/ψ, $\mathrm{dBR}(J/\psi, S)/\mathrm{d}m_{\pi\pi}$, as a function of $m_{\pi\pi}$ in units of $10^{-5}\,\mathrm{GeV}^{-1}$ is shown. We use $\alpha_s(\mu) = 0.7$ and $M_2^G(\mu) = 0.5$ as used in [17]. The solid line denotes the distribution by using (30), while the dashed line denotes the distribution using $\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle$ at the leading order of chiral perturbation theory. Integrating over $2m_\pi \le m_{\pi\pi} \le 0.70\,\mathrm{GeV}$, the decay branching ratios for J/ψ in the considered kinematic region are 1.8×10^{-5} and 3.5×10^{-5}, by using the result at leading and next-to-leading order of chiral perturbation theory respectively, indicating the importance of the next-to-leading order chiral corrections to the matrix element of the QCD total energy-momentum tensor. It should be noted all our numerical results are insensitive to the values of $M_2^G(\mu)$ and $\alpha_s(\mu)$; by varying the value of $M_2^G(\mu)$ from 0 to its asymptotic value (22), all numerical results are changed by less than 20%. Since $\alpha_s(\mu)$ appears always with $M_2^G(\mu)$ in the form $\alpha_s(\mu)\cdot M_2^G(\mu)$, the same is also true for $\alpha_s(\mu)$. Our results indicate that this decay mode and the $m_{\pi\pi}$ distribution are observable at BES II and at the proposed BES III and CLEO-C. Experimental study of the decay can test our model for $h(\tau, k)$ or extract it. This will provide information on how gluons are converted into two soft pions.

Although we have made the Ansatz for the function $h(\tau, k)$ in (26), where the shape as a function of τ is fixed, and the parameter $a(k)$ is just a normalization factor determined by the matrix element $\langle\pi^+\pi^-|\alpha_s\boldsymbol{E}^a(0)\cdot\boldsymbol{E}^a(0)|0\rangle$, but we can expect that our results for the branching ratio will be not changed dramatically with a change of the shape, because the normalization factor is fixed.

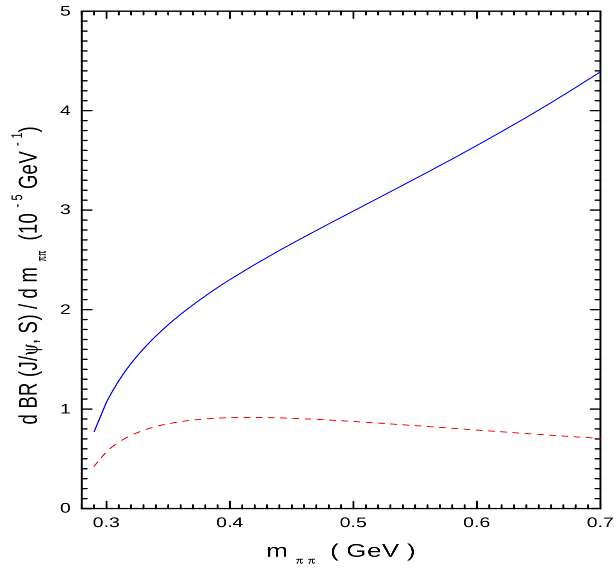

Fig. 8. The differential decay branching ratio of J/ψ, dBR $(J/\psi, S)/\mathrm{d}m_{\pi\pi}$, as a function of $m_{\pi\pi}$ in units of $10^{-5}\,\mathrm{GeV}^{-1}$ with the cuts. The dashed line denotes the distribution by using the leading order result of chiral perturbation theory for $\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle$, while the solid line denotes the distribution by adding one-loop correction to the matrix element

The corresponding $m_{\pi\pi}$ distribution for $\Upsilon(1S)$, referred to as $\mathrm{dBR}(\Upsilon, S)/\mathrm{d}m_{\pi\pi}$, in units of $10^{-6}\,\mathrm{GeV}^{-1}$, with $0 \le |\boldsymbol{k}| \le (1/10)M_\Upsilon$ and $2m_\pi < m_{\pi\pi} < 0.7\,\mathrm{GeV}$, is shown in Fig. 9. The solid line denotes the distribution using next-to-leading order chiral perturbative theory to determine the matrix element $\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle$, while the dashed line denotes the distribution using leading order chiral perturbative theory for this matrix element. Integrating over $2m_\pi \le m_{\pi\pi} \le 0.70\,\mathrm{GeV}$, the decay branching ratios for $\Upsilon(1S)$ in the considered kinematic region are 1.5×10^{-6} and 3.5×10^{-6}, by using the result at leading and next-to-leading order of chiral perturbation theory respectively. With the numerical results the decay mode may be difficult to be observed even at CLEO-C. However, we can learn from comparing Figs. 8 and 9 that when the phase space becomes larger, the next-to-leading order chiral corrections to the matrix element $\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle$ change the shape of $m_{\pi\pi}$ distribution dramatically.

4 Decays of B_c into a lepton pair combined with two pions

The observation of the meson B_c via the decay mode $B_c^\pm \to J/\psi + \ell^\pm\nu$ has been reported recently by the Collider Detector at Fermilab (CDF) Collaboration [19]. The B_c^+ meson is the lowest-mass bound state containing a

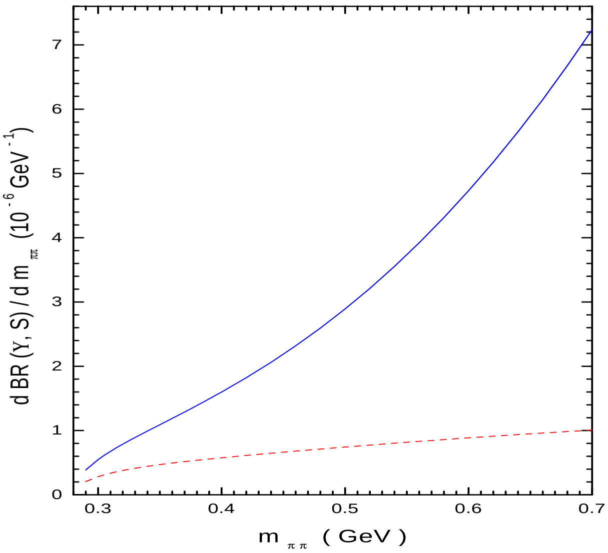

Fig. 9. The differential decay branching ratio of $\Upsilon(1S)$, referred to as $\mathrm{dBR}(\Upsilon, S)/\mathrm{d}m_{\pi\pi}$, as a function of $m_{\pi\pi}$ in units of $10^{-6}\,\mathrm{GeV}^{-1}$. The dashed line denotes the distribution by using the leading order result of chiral perturbation theory for $\langle\pi^+\pi^-|\theta^\mu_\mu|0\rangle$, while the solid line denotes the distribution by adding one-loop correction to the matrix element

charm quark and a bottom antiquark. It has non-zero flavor and can decay only via the weak interaction. Hence it has a very long lifetime, $\tau(B_c^+) = 0.46^{+0.18}_{-0.16}(\mathrm{stat.}) \pm 0.03(\mathrm{syst.})$ps. It will offer a new window for studying the weak decay mechanism of heavy flavors and test various non-perturbative models for bound states. The leptonic decay of B_c to one heavy meson has been studied in various models [20,21]. In this section we study the leptonic decay of B_c^+ into two pions. The first part of this section is devoted to leptonic decay of B_c^+ into two hard pions; the decay into two soft pions is studied in the second part.

4.1 The leptonic decays of B_c^+ combinedwith two hard pions

We study this exclusive decay in the rest frame of B_c^+:

$$B_c^+(P) \to l^+(p_1) + \nu_l(p_2) + \pi^+(k_{\pi^+}) + \pi^-(k_{\pi^-}), \quad (31)$$

where $l = e, \mu$, and the momenta are indicated in the brackets. We study the decay in the region where the two-pion state has a small invariant mass and has a large total momentum. Similarly as in Sect. 2 the decay amplitude can be factorized; the non-perturbative effect to the B_c^+ meson is then represented by a NRQCD matrix element, and that related to the two pions is represented by the same distribution amplitude of two gluons in the isoscalar pion pair, $\Phi^G(x_1, \zeta, m_{\pi\pi})$, which is defined in (13). The S-matrix element for the decay is

$$\langle f|S|i\rangle = \mathrm{i}\frac{G_\mathrm{F}}{\sqrt{2}} V_{bc} L_\mu \cdot \int \mathrm{d}^4 z \mathrm{e}^{\mathrm{i}q\cdot z} \times \langle\pi^+\pi^-|\bar{b}(z)\gamma^\mu(1-\gamma^5)c(z)|B_c^+\rangle, \quad (32)$$

where V_{bc} is the Cabibbo–Kobayashi–Maskawa matrix element, $c(z)$ and $\bar{b}(z)$ is the Dirac field for c-quark and for b-quark respectively, $q = p_1 + p_2$, and

$$L_\mu = \bar{u}(p_2)\gamma_\mu(1-\gamma^5)v(p_1); \quad (33)$$

$\bar{u}(p_2)$ and $v(p_1)$ are the spinors of the leptons. Using the method in Sect. 2, keeping leading terms in the heavy quark expansion and in the velocity expansion, we have

$$\begin{aligned}\langle f|S|i\rangle = &\frac{\mathrm{i}G_\mathrm{F}}{24\sqrt{2}} V_{bc} g_s^2 (2\pi)^4 \delta^4(P-k-q) L^\mu \cdot \\ &\times \langle 0|\chi_b^\dagger\psi_c|B_c^+\rangle \\ &\times \int_0^1 \mathrm{d}x_1 \frac{\Phi^G(x_1,\zeta,m_{\pi\pi})}{x_1(1-x_1)} \cdot \\ &\times \left[\frac{1}{2} d_\mathrm{T}^{\mu_1\nu_1} \cdot H_{\mu\mu_1\nu_1}(P,k,k_1)\right], \end{aligned} \quad (34)$$

where $\chi_b^\dagger(\psi_c)$ is the NRQCD field for the $\bar{b}(c)$-quark, and $H_{\mu\mu_1\nu_1}(P,k,k_1)$ is the hard part of the decay amplitude and can be calculated perturbatively. We obtain

$$\frac{1}{2} d_\mathrm{T}^{\mu_1\nu_1} \cdot H_{\mu\mu_1\nu_1}(P,k,k_1) = \frac{8M_{Bc}P^\mu}{(M_{Bc}^2 - q^2)m_b m_c}, \quad (35)$$

where $L_\mu q^\mu = 0$ for $m_l = 0$ is used. The differential decay width can be written as

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^8} \cdot \frac{\pi^2}{32} \cdot \frac{|\boldsymbol{k}|}{M_\psi} \cdot \beta\beta_l' \overline{\sum} |M|^2 \mathrm{d}q^2 \mathrm{d}m_{\pi\pi}^2 \mathrm{d}\cos\theta_\pi \mathrm{d}\cos\theta_l \mathrm{d}\phi, \quad (36)$$

where $\beta_l' = 1 - m_l^2/q^2$ is the velocity of l^+ in the center of mass frame of $l^+\nu_l$.

To present numerical predictions, the NRQCD matrix element $\langle 0|\chi_b^\dagger\psi_c|B_c^+\rangle$ should be known. It is related to the B_c decay constant f_{Bc} via

$$|\langle 0|\chi_b^\dagger\psi_c|B_c^+\rangle|^2 = \frac{1}{2} f_{Bc}^2 M_{Bc}, \quad (37)$$

with $f_{Bc} = 480\,\mathrm{MeV}$ taken from [20]. The other parameters take the following values: $M_{Bc} = 6.4\,\mathrm{GeV}, |V_{bc}| = 4.0\times10^{-2}, G_\mathrm{F} = 1.166\times10^{-5}\,\mathrm{GeV}^{-2}, \alpha_\mathrm{s}(M_{Bc}) = 0.24$. We use the cuts $k^+ \geq 10k^-$, $k^0 + \boldsymbol{k} \geq 2.0\,\mathrm{GeV}$ and $2m_\pi \leq m_{\pi\pi} \leq 0.70\,\mathrm{GeV}$.

With these parameters and cuts we can predict the differential decay branching ratio in the considered region. The $m_{\pi\pi}$ distribution of the B_c^+ semileptonic decay, $\mathrm{dBR}(B_c, H)/\mathrm{d}m_{\pi\pi}$, in units of $10^{-7}\,\mathrm{GeV}^{-1}$ with the cuts is shown in Fig. 10, and the q^2 distribution is presented in Fig. 11. Since the absolute squared matrix element of the B_c^+ decay in this region is almost independent of q^2, the shape of the q^2 distribution is determined mainly by the phase space factors. The decay branching ratio is 5.1×10^{-7}; belonging to it is the s-wave contribution 4.7×10^{-7}. The estimated branching ratio shows that the decay mode in this region will be not observable even at the Large Hadron Collier (LHC).

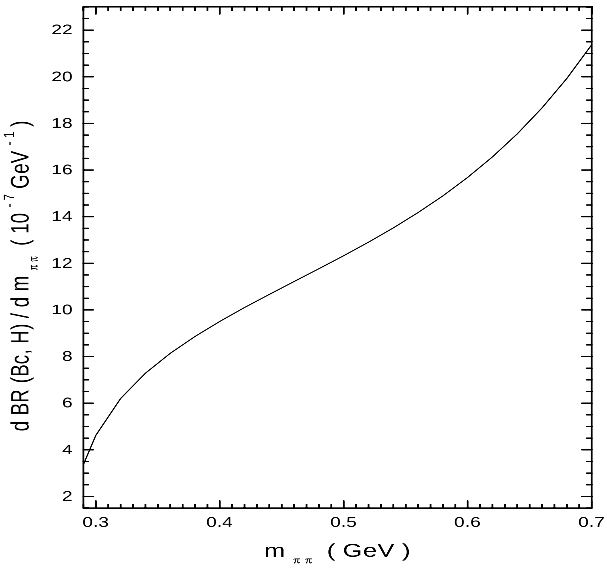

Fig. 10. The differential decay branching ratio of B_c^+, dBR $(B_c, H)/\mathrm{d}m_{\pi\pi}$, as a function of $m_{\pi\pi}$ in units of $10^{-7}\,\mathrm{GeV}^{-1}$ with the cuts

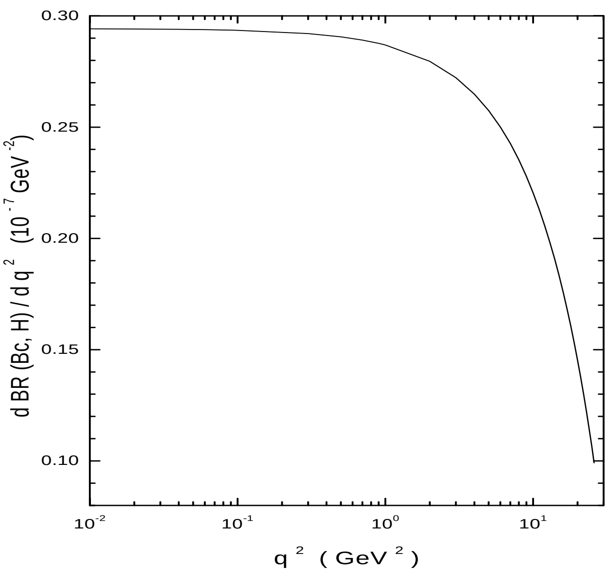

Fig. 11. The differential decay branching ratio of B_c^+, dBR $(B_c, H)/\mathrm{d}q^2$, as a function of q^2 in units of $10^{-7}\,\mathrm{GeV}^{-2}$ with the cuts

4.2 The leptonic decay of B_c^+ combined with two soft pions

In this subsection, we study the leptonic decay of B_c^+ combined with two soft pions. We use the same notation for momenta as before. With the method in [10] it is straightforward to obtain the S-matrix for the decay:

$$\langle f|S|i\rangle_s = \frac{\mathrm{i}G_\mathrm{F}}{3\sqrt{2}} V_{bc}(2\pi)^4\delta^4(P-k-q)\langle 0|\chi_b^\dagger\psi_c|B_c^+\rangle \times \frac{L_\mu \cdot P^\mu}{m_b m_c (k^0)^2} T_{\pi\pi}(k), \qquad (38)$$

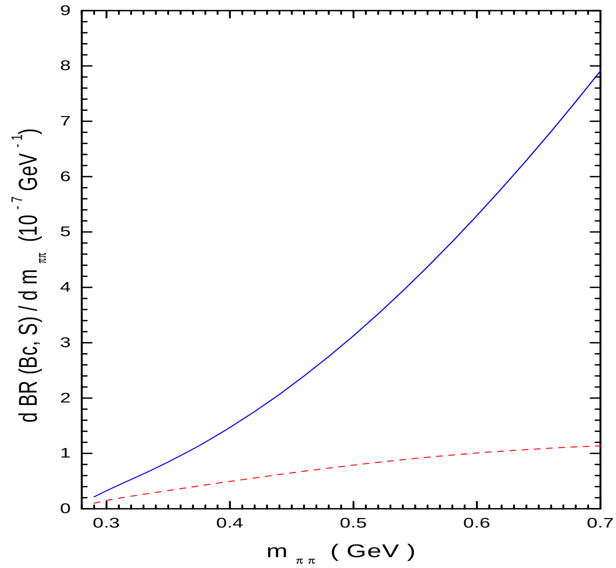

Fig. 12. The $m_{\pi\pi}$ distribution of the B_c^+ semileptonic decay to a pion pair, referred to as $\mathrm{dBR}(J/\psi, S)/\mathrm{d}m_{\pi\pi}$, in units of $10^{-7}\,\mathrm{GeV}^{-1}$ with the cuts. regel1220 The dashed line denotes the distribution by using the leading order result of chiral perturbation theory for $\langle\pi^+\pi^-|\theta_\mu^\mu|0\rangle$, while the solid line denotes the distribution by adding the one-loop correction to the matrix element

where $T_{\pi\pi}(k)$ is defined in (24); with our model for $h(\tau, k)$ given in Sect. 3, it can be expressed as

$$T_{\pi\pi}(k) = \frac{5\pi}{k^0}\langle\pi^+\pi^-|\alpha_\mathrm{s}\boldsymbol{E}^a(0)\cdot\boldsymbol{E}^a(0)|0\rangle. \qquad (39)$$

With this S-matrix element, it is straightforward to obtain the $m_{\pi\pi}$ distribution of the B_c^+ decay to two soft pions, which is shown in Fig. 12. The following cuts are used: $0 \le |\boldsymbol{k}| \le \frac{1}{10}M_{Bc}$ and $2m_\pi \le m_{\pi\pi} \le 0.70\,\mathrm{GeV}$. In Fig. 12, the solid line represents the $m_{\pi\pi}$ distribution by using next-to-leading order chiral perturbative theory to determine the matrix element $\langle\pi^+\pi^-|\theta_\mu^\mu|0\rangle$, while the dashed line denotes the distribution by using leading order chiral perturbative theory for this matrix element. Integrating over $2m_\pi \le m_{\pi\pi} \le 0.70\,\mathrm{GeV}$, the decay branching ratios for B_c^+ in the kinematic region are 3.6×10^{-7} and 1.5×10^{-7}, by using the result at leading and next-to-leading order of chiral perturbation theory respectively. The numerical results show that the decay mode is not observable even at LHC. But we can learn from Figs. 8, 12 and 9 that when the phase spaces become larger, the next-to-leading order chiral corrections to the matrix element $\langle\pi^+\pi^-|\theta_\mu^\mu|0\rangle$ change the shape of the $m_{\pi\pi}$ distribution dramatically.

5 Summary

In this paper we have studied the exclusive decay of J/ψ, Υ, and B_c into a lepton pair combined with two pions, where the two pions can be soft or hard with a small invariant mass. In both cases the decay amplitude can be factorized, in which the non-perturbative effect related to the heavy meson is represented by a NRQCD matrix element, and that related to the two pions is represented by a distribution amplitude of two gluons in the isoscalar pion pair in the case with hard pions, and by a correlator of chromoelectric fields in the case with soft pions. With suitable models for gluon conversion into soft or hard pions we are able to predict the branching ratios and various distributions.

Our numerical results show that the leptonic decay of J/ψ combined with two hard pions or with two soft pions can be observed at BES II and at the proposed BES III and CLEO-C, while the other decays have a too small branching ratio to be observed. If the decays of J/ψ are observed in experiment, it will provide information on how gluons, which are fundamental degrees of freedom in QCD, are converted into observed pions.

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